# $B$ meson light-cone wavefunctions in the heavy quark limit*

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We present a systematic study of the $B$ meson light-cone wavefunctions in QCD in the heavy-quark limit. We construct model-independent formulae for the light-cone wavefunctions in terms of independent dynamical degrees of freedom, which exactly satisfy the QCD equations of motion and constraints from heavy-quark symmetry. The results demonstrate novel behaviors of longitudinal as well as transverse momentum distribution in the $B$ mesons.

Recently systematic methods based on the QCD factorization have been developed for the exclusive $B$ meson decays into light mesons [1] (see also Ref. [2]). Essential ingredients in this approach are the light-cone distribution amplitudes for the participating mesons, which express nonperturbative long-distance contribution to the factorized amplitudes. The light-cone distribution amplitudes describe the probability amplitudes to find the meson in a state with the constituents carrying definite light-cone momentum fraction, and thus are process-independent quantity. For the light mesons ($\pi$, $K$, $\rho$, $K^*$, etc.) appearing in the final state, systematic model-independent study of the light-cone distributions exists for both leading and higher twists [3]. On the other hand, the light-cone distribution amplitudes for the $B$ mesons are not well-known at present and they provide a major source of uncertainty in the calculations of the decay rates.

By definition, the distribution amplitudes are obtained from light-cone wavefunctions at (almost) zero transverse separation of the constituents, $\phi(x) \sim \int_{k_T^2<\mu^2} d^2k_T \Phi(x, \boldsymbol{k}_T)$. The light-cone wavefunctions with transverse momentum dependence are also necessary for computing the power corrections to the exclusive amplitudes, and for estimating the transition form factors for $B \to D$, $B \to \pi$, etc, which constitute another type of long-distance contributions appearing in the factorization approaches for the exclusive $B$ meson decays.

In this work [4,5], we demonstrate that, in the heavy-quark limit relevant for the factorization approaches for the exclusive $B$ meson decays, the $B$ meson light-cone wavefunctions obey exact differential equations, which are based on heavy-quark symmetry and the QCD equations of motion. As solution of those differential equations, we derive the model-independent formulae for the light-cone wavefunctions, which involve not only the leading Fock-states with a minimal number of (valence) partons but also the higher Fock-states with additional dynamical gluons.

The light-cone wavefunctions are related to the usual Bethe-Salpeter wavefunctions at equal light-cone time $z^+ = (z^0 + z^3)/\sqrt{2}$. In the heavy-quark limit, the quark-antiquark light-cone wavefunctions $\tilde{\Phi}_\pm(t, z^2)$ of the $B$ mesons can be introduced in terms of vacuum-to-meson matrix element of nonlocal operators in the the heavy-quark

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effective theory (HQET) [6,5]:

$$\langle 0|\bar{q}(z)\Gamma h_v(0)|\bar{B}(p)\rangle = -\frac{if_B M}{2}\mathrm{Tr}\left[\gamma_5\Gamma\frac{1+\not{v}}{2}\right.$$
$$\left.\times\left\{\tilde{\Phi}_+(t,z^2) - \not{z}\frac{\tilde{\Phi}_+(t,z^2)-\tilde{\Phi}_-(t,z^2)}{2t}\right\}\right]. \quad (1)$$

Here $z^\mu = (0, z^-, \boldsymbol{z}_T)$, $z^2 = -\boldsymbol{z}_T^2$, $v^2 = 1$, $t = v\cdot z$, and $p^\mu = Mv^\mu$ is the 4-momentum of the $B$ meson with mass $M$. $h_v(x)$ denotes the effective $b$-quark field, $b(x) \approx \exp(-im_b v\cdot x)h_v(x)$, and is subject to the on-shell constraint, $\not{v}h_v = h_v$ [7]. $\Gamma$ is a generic Dirac matrix and, here and in the following, the path-ordered gauge factors are implied in between the constituent fields. $f_B$ is the decay constant defined as usually as $\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 h_v(0)|\bar{B}(p)\rangle = if_B M v^\mu$, so that $\tilde{\Phi}_\pm(t=0, z^2=0) = 1$. Eq. (1) is the most general parameterization compatible with Lorentz invariance and the heavy-quark limit.

Higher Fock components in the $B$ mesons are described by multi-particle wavefunctions. We explicitly deal with quark-antiquark-gluon three-particle wavefunctions, defined as [4]

$$\langle 0|\bar{q}(z)\, gG_{\mu\nu}(uz)\, z^\nu\,\Gamma\, h_v(0)|\bar{B}(p)\rangle$$
$$= \frac{1}{2} f_B M\,\mathrm{Tr}\left[\gamma_5\,\Gamma\,\frac{1+\not{v}}{2}\left\{(v_\mu\,\not{z} - t\,\gamma_\mu)\right.\right.$$
$$\times\left(\tilde{\Psi}_A(t,u) - \tilde{\Psi}_V(t,u)\right) - i\,\sigma_{\mu\nu}z^\nu\,\tilde{\Psi}_V(t,u)$$
$$\left.\left. - z_\mu\,\tilde{X}_A(t,u) + \frac{z_\mu}{t}\,\not{z}\tilde{Y}_A(t,u)\right\}\right] + \ldots, \quad (2)$$

where the ellipses stand for the terms which involve one or more powers of $z^2$ and are irrelevant for the present work. We have the four functions $\tilde{\Psi}_V, \tilde{\Psi}_A, \tilde{X}_A$ and $\tilde{Y}_A$ as the independent three-particle wavefunctions in the heavy-quark limit.

The QCD equations of motion impose a set of relations between the above wavefunctions [4,5]. They can be derived most directly from the exact identities between the nonlocal operators:

$$\frac{\partial}{\partial x^\mu}\bar{q}(x)\gamma^\mu\Gamma h_v(0)$$
$$= i\int_0^1 du u\; \bar{q}(x)gG_{\mu\nu}(ux)x^\nu\gamma^\mu\Gamma h_v(0)\;, \quad (3)$$
$$v^\mu\frac{\partial}{\partial x^\mu}\bar{q}(x)\Gamma h_v(0)$$
$$= i\int_0^1 du(u-1)\;\bar{q}(x)gG_{\mu\nu}(ux)v^\mu x^\nu\Gamma h_v(0)$$
$$+v^\mu\,\frac{\partial}{\partial y^\mu}\;\bar{q}(x+y)\Gamma h_v(y)\Big|_{y\to 0}\;, \quad (4)$$

where $G_{\mu\nu}$ is the gluon field strength tensor, and we have used the equations of motion $\not{D}q = 0$ and $v\cdot Dh_v = 0$ with $D_\mu = \partial_\mu - igA_\mu$ the covariant derivative. Taking the matrix element with $x_\mu \to z_\mu$, the LHS of these identities yield $\tilde{\Phi}_+(t,z^2)$, $\tilde{\Phi}_-(t,z^2)$ defined in Eq. (1) and their derivatives, $\partial\tilde{\Phi}_\pm(t,z^2)/\partial t$ and $\partial\tilde{\Phi}_\pm(t,z^2)/\partial z^2$. The terms in the RHS, which are given by integral of quark-antiquark-gluon operator, are expressed by the three-particle wavefunctions of Eq. (2). The last term of Eq. (4), the derivative over the total translation, yields $\tilde{\Phi}_\pm(t,z^2)$ multiplied by

$$\bar{\Lambda} = M - m_b = \frac{iv\cdot\partial\langle 0|\bar{q}\Gamma h_v|\bar{B}(p)\rangle}{\langle 0|\bar{q}\Gamma h_v|\bar{B}(p)\rangle}\;. \quad (5)$$

This is the usual "effective mass" of meson states in the HQET [7].

Substituting all the Dirac matrices for $\Gamma$, we obtain the four independent constraint equations between the two- and three-particle wavefunctions from Eqs. (3) and (4). We solve this system of equations for the relevant two cases: (i) In the light-cone limit $z^2 \to 0$ ($\boldsymbol{z}_T \to 0$), with fully taking into account the contribution due to the three-particle wavefunctions. The solution gives exact model-independent representaions for the light-cone distribution amplitudes in terms of independent dynamical degrees of freedom. (ii) For $z^2 \neq 0$ but in the approximation neglecting the contribution of the three-particle wavefunctions. The solution gives exact analytic formulae for the light-cone wavefunctions with transverse momentum dependence within the valence Fock-states.

Now we discuss the case (i) in detail [4]. In the light-cone limit, the light-cone wavefunctions of Eq. (1) reduce to the light-cone distribution amplitudes $\tilde{\phi}_\pm(t)$ as [8]

$$\tilde{\phi}_\pm(t) = \tilde{\Phi}_\pm(t, z^2 = 0)\;, \quad (6)$$

and we also introduce the shorthand notations, $\tilde{\phi}'_\pm(t) \equiv d\tilde{\phi}_\pm(t)/dt$ and $\partial\tilde{\phi}_\pm(t)/\partial z^2 \equiv \partial\tilde{\Phi}_\pm(t,z^2)/\partial z^2|_{z^2\to 0}$, which denote the derivatives with respect to the longitudinal and transverse separations, respectively. The first identity (3) yields the two equations:

$$\tilde{\phi}'_-(t) - \frac{1}{t}\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) = 2t\int_0^1 du\, u\,(\tilde{\Psi}_A(t,u) - \tilde{\Psi}_V(t,u)) \ , \quad (7)$$

$$\tilde{\phi}'_+(t) - \tilde{\phi}'_-(t) - \frac{1}{t}\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) + 4t\frac{\partial\tilde{\phi}_+(t)}{\partial z^2} = 2t\int_0^1 du\, u\,(\tilde{\Psi}_A(t,u) + 2\,\tilde{\Psi}_V(t,u) + \tilde{X}_A(t,u)), \quad (8)$$

and similarly the second identity (4) yields

$$\tilde{\phi}'_+(t) - \frac{1}{2t}\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) + i\bar{\Lambda}\tilde{\phi}_+(t) + 2t\frac{\partial\tilde{\phi}_+(t)}{\partial z^2} = t\int_0^1 du\,(u-1)\,(\tilde{\Psi}_A(t,u) + \tilde{X}_A(t,u)) \ , \quad (9)$$

$$\tilde{\phi}'_+(t) - \tilde{\phi}'_-(t) + \left(i\bar{\Lambda} - \frac{1}{t}\right)\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) + 2t\left(\frac{\partial\tilde{\phi}_+(t)}{\partial z^2} - \frac{\partial\tilde{\phi}_-(t)}{\partial z^2}\right) = 2t\int_0^1 du\,(u-1)\,(\tilde{\Psi}_A(t,u) + \tilde{Y}_A(t,u)). \quad (10)$$

These Eqs. (7)-(10) are exact in QCD in the heavy-quark limit.

Important observation is that we can eliminate the term $\partial\tilde{\phi}_+(t)/\partial z^2$ by combining Eqs. (8) and (9). The resulting equation, combined with Eq. (7), gives a system of two differential equations which involve the degrees of freedom along the light-cone only. By going over to the momentum space by $\tilde{\phi}_\pm(t) = \int d\omega\ e^{-i\omega t}\phi_\pm(\omega)$, and $\tilde{F}(t,u) = \int d\omega d\xi\ e^{-i(\omega+\xi u)t}F(\omega,\xi)$ with $F = \{\Psi_V, \Psi_A, X_A, Y_A\}$, the corresponding differential equations read [4]

$$\omega\frac{d\phi_-(\omega)}{d\omega} + \phi_+(\omega) = I(\omega) \ , \quad (11)$$

$$\left(\omega - 2\bar{\Lambda}\right)\phi_+(\omega) + \omega\phi_-(\omega) = J(\omega) \ , \quad (12)$$

where $I(\omega)$ and $J(\omega)$ denote the "source" terms due to three-particle wavefunctions as

$$I(\omega) = 2\frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\frac{\partial}{\partial\xi}\left[\Psi_A(\rho,\xi) - \Psi_V(\rho,\xi)\right],$$

$$J(\omega) = -2\frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\left[\Psi_A(\rho,\xi) + X_A(\rho,\xi)\right] - 4\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\frac{\partial\Psi_V(\rho,\xi)}{\partial\xi} \ . \quad (13)$$

Eqs. (11), (12) can be solved for $\phi_+(\omega)$ and $\phi_-(\omega)$. The boundary conditions are specified as $\phi_\pm(\omega) = 0$ for $\omega < 0$ or $\omega \to \infty$, because $\omega v^+$ has the meaning of the light-cone projection $k^+$ of the light-antiquark momentum in the $B$ meson, and the normalization condition is $\int_0^\infty d\omega\phi_\pm(\omega) = \tilde{\Phi}_\pm(0,0) = 1$. Obviously, the solution can be decomposed into two pieces as

$$\phi_\pm(\omega) = \phi_\pm^{(W)}(\omega) + \phi_\pm^{(g)}(\omega) \ , \quad (14)$$

where $\phi_\pm^{(W)}(\omega)$ are the solution with $I(\omega) = J(\omega) = 0$, which corresponds to the "Wandzura-Wilczek approximation [3]" $\Psi_V = \Psi_A = X_A = Y_A = 0$. $\phi_\pm^{(g)}(\omega)$ denote the pieces induced by the source terms $I(\omega)$ and $J(\omega)$.

We are able to obtain the analytic solution for the Wandzura-Wilczek part as

$$\phi_\pm^{(W)}(\omega) = \frac{\bar{\Lambda} \pm (\omega - \bar{\Lambda})}{2\bar{\Lambda}^2}\theta(\omega)\theta(2\bar{\Lambda} - \omega) \ . \quad (15)$$

Moreover, the solution for $\phi_\pm^{(g)}$ can be obtained straightforwardly, and reads ($\omega \geq 0$):

$$\phi_+^{(g)}(\omega) = \frac{\omega}{2\bar{\Lambda}}\mathcal{G}(\omega) \ , \quad (16)$$

$$\phi_-^{(g)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}}\mathcal{G}(\omega) + \frac{J(\omega)}{\omega} \ , \quad (17)$$

$$\mathcal{G}(\omega) = \theta(2\bar{\Lambda} - \omega)\left\{\int_0^\omega d\rho\frac{K(\rho)}{2\bar{\Lambda} - \rho} - \frac{J(0)}{2\bar{\Lambda}}\right\} - \theta(\omega - 2\bar{\Lambda})\int_\omega^\infty d\rho\frac{K(\rho)}{2\bar{\Lambda} - \rho} - \int_\omega^\infty d\rho\left(\frac{K(\rho)}{\rho} + \frac{J(\rho)}{\rho^2}\right) \ , \quad (18)$$

with $K(\rho) = I(\rho) + \left[1/(2\bar{\Lambda}) - d/d\rho\right]J(\rho)$. The solution (14) with Eqs. (15)-(18) is exact, and reveals that $\phi_\pm$ contain the three-particle contributions. This is also visualized explicitly in terms of the Mellin moments $\langle\omega^n\rangle_\pm \equiv \int d\omega\ \omega^n\phi_\pm(\omega)$ ($n = 0, 1, 2, \cdots$). Here we present some examples for a few low moments: $\langle\omega\rangle_\pm = (3 \pm 1)\bar{\Lambda}/3$, and

$$\langle\omega^2\rangle_\pm = \frac{4 \pm 2}{3}\bar{\Lambda}^2 + \frac{1}{3}\lambda_E^2 + \frac{1}{3}\lambda_H^2 \pm \frac{1}{3}\lambda_E^2 \ , \quad (19)$$

where $\lambda_E$ and $\lambda_H$ are due to $\phi_\pm^{(g)}$, and are related to the chromoelectric and chromomagnetic fields in the $B$ meson rest frame as $\langle 0|\bar{q}g\boldsymbol{E}\cdot\boldsymbol{\alpha}\gamma_5 h_v|\bar{B}(\boldsymbol{p}=0)\rangle = f_B M\lambda_E^2$, $\langle 0|\bar{q}g\boldsymbol{H}\cdot\boldsymbol{\sigma}\gamma_5 h_v|\bar{B}(\boldsymbol{p}=0)\rangle = if_B M\lambda_H^2$. Our solution (14) allows us to further derive the analytic formulae for the general moment $n$ in terms of matrix element of local two- and three-particle operators with dimension $n+3$ (see Ref. [4] for the detail).

The behavior of Eq. (1) for a fast-moving meson, $t = v\cdot z \to \infty$, shows that $\phi_+$ is of leading-twist whereas $\phi_-$ has subleading twist; the three-particle contributions to the leading-twist $\phi_+$ are in contrast with the case of the light mesons [3], where the leading-twist amplitudes correspond to the valence Fock component, while the higher-twist amplitudes involve multi-particle states. We note that there exists an estimate $\lambda_E^2/\bar{\Lambda}^2 = 0.36\pm 0.20$, $\lambda_H^2/\bar{\Lambda}^2 = 0.60\pm 0.23$ by QCD sum rules [8] (see Eq. (19)). This might suggest that, in the $B$ mesons, the three-particle contributions could play important roles even in the leading twist level. In this connection, the shape of our Wandzura-Wilczek contributions (15), which are determined uniquely in analytic form in terms of $\bar{\Lambda}$, is rather different from various "model" distribution amplitudes that have been used in the existing literature: One exapmle of such models is $\phi_+^{GN}(\omega) = (\omega/\omega_0^2)e^{-\omega/\omega_0}$ and $\phi_-^{GN}(\omega) = (1/\omega_0)e^{-\omega/\omega_0}$ with $\omega_0 = 2\bar{\Lambda}/3$, inspired by the QCD sum rule estimates [8]. These have very different shape compared with Eq. (15), except the behavior $\phi_+^{GN}(\omega)\sim\omega$, $\phi_-^{GN}(\omega)\sim$ const, as $\omega\to 0$.

Next we proceed to the case (ii), where we get

$$\omega\frac{\partial\Phi_-}{\partial\omega} + \Phi_+ + z^2\frac{\partial}{\partial z^2}\left(\Phi_+ - \Phi_-\right) = 0 \ , \qquad (20)$$

$$\left(\omega\frac{\partial}{\partial\omega}+2\right)\left(\Phi_+ - \Phi_-\right) + 4\frac{\partial^2}{\partial\omega^2}\frac{\partial\Phi_+}{\partial z^2} = 0 \ , \quad (21)$$

$$\left[(\omega-\bar{\Lambda})\frac{\partial}{\partial\omega}+\frac{3}{2}\right]\Phi_+ - \frac{1}{2}\Phi_- + 2\frac{\partial^2}{\partial\omega^2}\frac{\partial\Phi_+}{\partial z^2} = 0, \quad (22)$$

$$\left[(\omega-\bar{\Lambda})\frac{\partial}{\partial\omega}+2\right]\left(\Phi_+ - \Phi_-\right) + 2\frac{\partial^2}{\partial\omega^2}\left(\frac{\partial\Phi_+}{\partial z^2} - \frac{\partial\Phi_-}{\partial z^2}\right) = 0 \ , \quad (23)$$

corresponding to Eqs. (7), (8), (9), (10), respectively. Here Eqs. (20)-(23) are given in the "$\omega$-representation" $\Phi_\pm \equiv \Phi_\pm(\omega, z^2)$, instead of the "$t$-representation", via $\tilde{\Phi}_\pm(t, z^2) = \int d\omega\ e^{-i\omega t}\Phi_\pm(\omega, z^2)$. The light-cone limit is not taken so that the terms proportional to $z^2$ appear in Eq. (20). Note that we have neglected the contribution from the quark-antiquark-gluon three-particle operators.

By combining Eqs. (21) and (22), we eliminate the last term in their LHS. The resulting equation can be integrated with boundary conditions $\Phi_\pm(\omega, z^2) = 0$ for $\omega < 0$ or $\omega\to\infty$ as

$$\left(\omega - 2\bar{\Lambda}\right)\Phi_+ + \omega\Phi_- = 0 \ . \qquad (24)$$

In the limit $z^2\to 0$, Eqs. (20) and (24) reduce to Eqs. (11) and (12) in the Wandzura-Wilczek approximation. The corresponding solution (15) serves as "boundary conditions" to solve Eqs. (20)-(23) for $z^2\neq 0$. Then, from Eqs. (20) and (24), we find, as the solution in the Wandzura-Wilczek approximation for $z^2\neq 0$,

$$\Phi_\pm^{(W)}(\omega, z^2) = \phi_\pm^{(W)}(\omega)\xi\left(z^2\omega(2\bar{\Lambda}-\omega)\right) \ . \qquad (25)$$

Here $\xi(x)$ is a function of a single variable $x$, and can be determined from a remaining differential equation (22) or (23) as $\xi(x) = J_0\left(\sqrt{-x}\right)$ where $J_0$ is a (regular) Bessel function. This result gives analytic solution for the light-cone wavefunctions with the transverse separation $\boldsymbol{z}_T^2 = -z^2$. The momentum-space wavefunctions $\Phi_\pm^{(W)}(\omega, \boldsymbol{k}_T)$, defined by $\tilde{\Phi}_\pm^{(W)}(t, -\boldsymbol{z}_T^2) = \int d\omega d^2k_T\ e^{-i\omega t + i\boldsymbol{k}_T\cdot\boldsymbol{z}_T}\Phi_\pm^{(W)}(\omega, \boldsymbol{k}_T)$, read

$$\Phi_\pm^{(W)}(\omega, \boldsymbol{k}_T) = \frac{\phi_\pm^{(W)}(\omega)}{\pi}\delta\left(\boldsymbol{k}_T^2 - \omega(2\bar{\Lambda}-\omega)\right) . \quad (26)$$

The result (26) gives exact description of the valence Fock components of the $B$ meson wavefunctions in the heavy-quark limit, and represent their transverse momentum dependence explicitly. These results show that the dynamics within the two-particle Fock states is determined solely in terms of a single nonperturbative parameter $\bar{\Lambda}$.

The transverse momentum distributions in the $B$ mesons have been unknown, so that various models have been used in the literature. Frequently used models assume complete separation (factorization) between the longitudinal and transverse momentum-dependence in the wavefunctions (see e.g. Refs. [2,6]). A typical example of such models [2] is given by $\Phi_\pm^{KLS}(\omega, \boldsymbol{k}_T) = N\omega^2(1-\omega)^2 e^{-\omega^2/(2\beta^2)} \times e^{-\boldsymbol{k}_T^2/(2\kappa^2)}$ with some constants $N, \beta$, and $\kappa$. Eq. (26) shows that transverse and longitudinal momenta are strongly correlated through the combination $\boldsymbol{k}_T^2/[\omega(2\bar{\Lambda}-\omega)]$, therefore the "factorization models" are not justified. We further note that many models assume Gaussian distribution for the $\boldsymbol{k}_T$-dependence as in $\Phi_\pm^{KLS}$. These models show strong dumping at large $|\boldsymbol{z}_T|$ as $\sim \exp\left(-\kappa^2 \boldsymbol{z}_T^2/2\right)$. In contrast, our wavefunctions (25) have slow-damping with oscillatory behavior as $\Phi_\pm^{(W)}(\omega, -\boldsymbol{z}_T^2) \sim \cos(|\boldsymbol{z}_T|\sqrt{\omega(2\bar{\Lambda}-\omega)} - \pi/4)/\sqrt{|\boldsymbol{z}_T|}$.

Finally, we can estimate the effects neglected in our solution (26). Inspecting the $t \to 0$ limit of Eqs. (8), (10), one immediately obtains the exact result for $\partial\tilde{\phi}_\pm(0)/\partial z^2$, which gives the first moment of $\boldsymbol{k}_T^2$ as [5]

$$\int d\omega d^2k_T\ \boldsymbol{k}_T^2 \Phi_\pm(\omega, \boldsymbol{k}_T) = \frac{2}{3}\left(\bar{\Lambda}^2 + \lambda_E^2 + \lambda_H^2\right), \quad (27)$$

with $\lambda_E$ and $\lambda_H$ of Eq. (19). Here $\Phi_\pm = \Phi_\pm^{(W)} + \Phi_\pm^{(g)}$ denote the total wavefunctions which include the higher Fock contributions $\Phi_\pm^{(g)}$ induced by the three-particle operators. From Eq. (26), we get $\int d\omega d^2k_T\ \boldsymbol{k}_T^2\Phi_\pm^{(W)}(\omega, \boldsymbol{k}_T) = 2\bar{\Lambda}^2/3$, so that the terms $2(\lambda_E^2 + \lambda_H^2)/3$ of Eq. (27) come from $\Phi_\pm^{(g)}$. The result (27), combined with a QCD sum rule estimate of $\lambda_E, \lambda_H$ mentioned below Eq. (19), suggests that the higher Fock contributions might considerably broaden the transverse momentum distribution. However, qualitative features discussed above, like non-factorization of longitudinal and transverse directions, "slow-damping" for transverse directions, etc., will be unaltered by the effects of multi-particle states.

To summarize, we have derived a system of differential equations for the $B$ meson light-cone wavefunctions and obtained the corresponding analytic solution. The differential equations are derived from the exact equations of motion of QCD in the heavy-quark limit. The heavy-quark symmetry plays an essential role by reducing the number of independent wavefunctions drastically, so that the configuration of quark and antiquark in the $B$ mesons is described by only two light-cone wavefunctions. As a result, a system of four differential equations from the equations of motion allows us to obtain model-independent formulae of these two wavefunctions, which reveal roles of the leading Fock-states, as well as the higher Fock-states with additional dynamical gluons. Also due to the power of heavy-quark symmetry, our Wandzura-Wilczek parts (15), (26), which correspond to the leading Fock-states, are given in simple analytic formulae involving one single nonperturbative parameter $\bar{\Lambda}$. Heavy-quark symmetry also guarantees that our solutions determine the light-cone wavefunctions for the $B^*$ mesons and also for the $D$, $D^*$ mesons in the heavy-quark limit.

We emphasize that our solutions provide the powerful framework for building up the $B$ meson light-cone wavefunctions and their phenomenological applications, because the solutions satisfy all relevant QCD constraints. Further developments like those required to clarify the effects of multi-particle states can be exploited systematically starting from the exact results in this work.

## REFERENCES


1. M. Beneke et al., Nucl. Phys. **B591** (2000) 313; **B606** (2001) 245; C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. **87** (2001) 201806; G. Buchalla, in these proceedings.
2. Y. Y. Keum, H. Li and A. I. Sanda, Phys. Rev. **D63** (2001) 054008.
3. V. M. Braun and I. E. Filyanov, *Z. Phys.* **C48**, 239 (1990); P. Ball et al., Nucl. Phys. **B529** (1998) 323.
4. H. Kawamura, J. Kodaira, C.F. Qiao and K. Tanaka, Phys. Lett. **B523** (2001) 111; Erratum-ibid. **B536** (2002) 344.
5. H. Kawamura, J. Kodaira, C.F. Qiao and K. Tanaka, hep-ph/0112174.
6. S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. **B625** (2002) 239.
7. M. Neubert, Phys. Rept. **245** (1994) 259.
8. A. G. Grozin and M. Neubert, Phys. Rev. **D55** (1997) 272.